Optics, Spring 2018

Exercise 2, 1.2.2018

Problem 2 is worth 2 points.

1. Snell’s law of refraction

a) Derive Snell’s law using Fermat’s principle.
b) A ray incident at $\theta_i$ to a planar glass plate exits the plate at $\theta_o$ (see the figure). Show that $\theta_i = \theta_o$. The plate is immersed in air.
c) Derive the expression for the displacement $a$ if the thickness of the plate is $d$.

![Diagram](image)

2. The Fresnel equations

Electromagnetic plane wave is incident at an interface between two dielectric media (refractive indices $n_1$ and $n_2$, assume that $\mu_r = 1$). The amplitude of the incident electric field $E_{0i}$ is known. Derive the Fresnel equations describing the reflected and transmitted electric field amplitudes $E_{0r}$ and $E_{0t}$ when the incident electric field is parallel to the plane of incidence.

**Hint:** Follow the description in Hecht, 4th edition, chapter 4.6.2. Explain every intermediate step. Do not just blindly copy the book chapter.

3. Reflectance and transmittance

a) Derive the expressions for the amplitude coefficients given by problem 2 for normal incidence.
b) Derive the expression for coefficients of transmittance and reflectance at normal incidence:

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$
$$T = \frac{4n_1n_2}{(n_2 + n_1)^2}$$
c) What is the percent of reflected irradiance at an air-glass interface (normal incidence, $n_{air} = 1, n_{glass} = 1.5$)?

4. Brewster’s angle

Light is incident at an interface between two dielectric media. The incident angle is called Brewster’s angle (a.k.a. the polarization angle) when the sum of incident and refracted angles is 90° ($\theta_i + \theta_r = 90°$).

a) Calculate the angle at an air-glass interface using Snell’s law.
b) Calculate the coefficient of reflected amplitude incident at Brewster’s angle when the electric field is polarized in the direction parallel to the plane of incidence.